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# Dilepton production near partonic threshold in transversely polarized $\bar{p}p$ collisions

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## Abstract

It has recently been suggested that collisions of transversely polarized protons and antiprotons at the GSI could be used to determine the nucleon's transversity densities from measurements of the double-spin asymmetry for the Drell-Yan process. We analyze the role of higher-order perturbative QCD corrections in this kinematic regime, in terms of the available fixed-order contributions as well as of all-order soft-gluon resummations. We find that the combined perturbative corrections to the individual unpolarized and transversely polarized cross sections are large. We trace these large enhancements to soft gluon emission near partonic threshold, and we suggest that with a physically-motivated cut-off enhancements beyond lowest order are moderated relative to resummed perturbation theory, but still significant. The unpolarized dilepton cross section for the GSI kinematics may therefore provide information on the relation of perturbative and nonperturbative dynamics in hadronic scattering. The spin asymmetry turns out to be rather robust, relatively insensitive to higher orders, resummation, and the cut-offs.

# 1 Introduction

The partonic structure of polarized nucleons at the leading-twist level is characterized by the unpolarized, longitudinally polarized, and transversely polarized parton distribution functions  $f$ ,  $\Delta f$ , and  $\delta f$ , respectively [1]. In contrast to the distributions  $f$  and  $\Delta f$ , we have essentially no knowledge from experiment so far about the transversity distributions  $\delta f$ , even though there are now first indications [2] that some of them are non-vanishing. The  $\delta f$  were first introduced in [3]. They are defined as [1, 3, 4, 5] the difference of probabilities for finding a parton of flavor  $f$  at scale  $\mu$  and light-cone momentum fraction  $x$  with its spin aligned ( $\uparrow\uparrow$ ) or anti-aligned ( $\downarrow\uparrow$ ) to that of the transversely polarized nucleon:

$$\delta f(x, \mu) \equiv f_{\uparrow\uparrow}(x, \mu) - f_{\downarrow\uparrow}(x, \mu) . \quad (1)$$

By virtue of factorization theorems [6, 7], the parton densities can be probed universally in a variety of inelastic scattering processes for which it is possible to separate (“factorize”) the long-distance physics relating to nucleon structure from a partonic short-distance scattering that can be calculated in QCD perturbation theory. It was realized a long time ago [1, 3, 4] that due to its chirally-odd structure, transversity decouples from inclusive deeply-inelastic scattering, but that inelastic collisions of two transversely polarized nucleons should offer good possibilities to access transversity. In particular, the Drell-Yan processes  $pp \rightarrow l^+l^-X$ ,  $p\bar{p} \rightarrow l^+l^-X$  ( $l = e, \mu$ ) were identified as promising sources of information on transversity [8, 9, 10]. This is so because there is no gluon transversity distribution at leading twist [1, 4]. For the Drell-Yan process, the lowest-order partonic process is  $q\bar{q} \rightarrow \gamma^*$ , with gluonic contributions to the unpolarized cross section in the denominator of the transverse double-spin asymmetry

$$A_{TT} = \frac{\sigma^{\uparrow\uparrow} - \sigma^{\uparrow\downarrow}}{\sigma^{\uparrow\uparrow} + \sigma^{\uparrow\downarrow}} \quad (2)$$

only arising as higher-order corrections. Therefore,  $A_{TT}$  may be sizable for the Drell-Yan process, in contrast to other hadronic processes such as high-transverse-momentum prompt photon and jet production [4, 11, 12, 13, 14] which in the unpolarized case are largely driven by gluons in the initial state and are hence expected to have a very suppressed  $A_{TT}$ .

Clean information on transversity should be gathered from polarized proton-proton collisions at the BNL Relativistic Heavy Ion Collider (RHIC) where the Drell-Yan process is a major focus [15]. In  $pp$  collisions, however, the Drell-Yan process probes products of valence quark and sea antiquark distributions. It is possible that antiquarks in the nucleon carry only little transverse polarization since, due to the absence of a gluon transversity, a source for the perturbative generation of transversity sea quarks from  $g \rightarrow q\bar{q}$  splitting is missing. In addition, at RHIC energies and for Drell-Yan masses of a few GeV, the partonic momentum fractions are fairly small, so that the denominator of  $A_{TT}$  is large due to the small- $x$  rise of the unpolarized sea quark distributions. Thus, even for the Drell-Yan process, the spin asymmetry  $A_{TT}$  at RHIC will probably be at most a few per cent, as theoretical studies have shown [9, 10, 16].

It has recently been proposed to add polarization to planned  $\bar{p}p$  collision experiments at the GSI, and to perform measurements of  $A_{TT}$  for the Drell-Yan process [17, 18, 19, 20]. This is a very exciting idea, since unique information on transversity could be obtained in this way. The results would be complementary to what can be obtained from RHIC measurements. First of all,

in  $\bar{p}p$  collisions the Drell-Yan process mainly probes products of two *quark* densities,  $\delta q \times \delta q$ , since the distribution of antiquarks in antiprotons equals that of quarks in the proton. In addition, kinematics in the proposed experiments are such that rather large partonic momentum fractions,  $x \sim 0.5$ , are probed. One therefore accesses the valence region of the nucleon. Estimates [19, 20, 21] for the GSI PAX and ASSIA experiments show that the expected spin asymmetry  $A_{TT}$  should be very large, of order 40% or more.

It is important, however, to keep in mind that the kinematic region to be accessed in the first stage of the GSI experiments, with Drell-Yan masses  $M$  of 1 – 4 GeV or so, but a center-of-mass energy of only  $\sqrt{S} \approx 5.3$  GeV for the baseline fixed-target program, is not really the “classic” regime where parton model ideas, factorization, and perturbative QCD are a priori expected to provide adequate descriptions. This is of course crucial since the interpretation of  $A_{TT}$  in terms of transversity relies exactly on these concepts. To be more precise, at high energies and large dilepton invariant mass  $M$  the cross section factorizes [6, 7] into convolutions of parton densities and perturbative partonic hard-scattering cross sections, as mentioned above. Schematically,

$$M^4 \frac{d\sigma}{dM^2} = \sum_{a,b} f_a \otimes f_b \otimes \frac{M^4 d\hat{\sigma}_{ab}}{dM^2} + \mathcal{O}\left(\frac{\lambda}{M}\right)^p. \quad (3)$$

For simplicity, we have considered here the unpolarized cross section, and we have also integrated over the rapidity of the lepton pair and only focused on the total Drell-Yan cross section. We also have not written out the precise form of the convolutions, which will be given below. For the moment, we are only interested in the important features visible in Eq. (3). The quantities one wants to determine from measurement of the left-hand-side of Eq. (3) are the parton distributions  $f_a, f_b$ . The partonic cross sections,  $\hat{\sigma}_{ab}$ , for the reactions  $ab \rightarrow \gamma^* X$  may be calculated in QCD perturbation theory. Their expansion in terms of the strong coupling constant  $\alpha_s(M)$  reads

$$d\hat{\sigma}_{ab} = d\hat{\sigma}_{ab}^{(0)} + \frac{\alpha_s(M)}{\pi} d\hat{\sigma}_{ab}^{(1)} + \left(\frac{\alpha_s(M)}{\pi}\right)^2 d\hat{\sigma}_{ab}^{(2)} + \dots, \quad (4)$$

corresponding to lowest order (LO), next-to-leading order (NLO), and so forth. The earlier studies [19, 20] for the Drell-Yan process at GSI energies used LO hard-scattering cross sections to estimate the expected spin asymmetries. Depending on kinematics, however, the higher-order corrections may be very important. As we will show below, this is the case for the planned GSI measurements. In addition, as indicated in Eq. (3), factorization of the hadronic cross section in terms of twist-2 distributions is of course not exact, but holds only to leading power in  $M$ . There are corrections to the (dimensionless) cross section  $M^4 d\sigma/dM^2$  that are down by inverse powers of the hard scale, that is, of the form  $(\lambda/M)^p$  with some  $p$  and some hadronic mass scale  $\lambda$  [22]. These power corrections will also depend on  $\tau = M^2/S$  and are generally expected to increase with increasing  $\tau$ . The measured spin asymmetry  $A_{TT}$  can only be reliably interpreted in terms of the transversity densities if the higher order and power corrections can either be shown to be small in the accessible kinematic domain, and/or if they are sufficiently well understood. The aim of this paper is to address primarily the question of how large the higher-order QCD corrections to the Drell-Yan process are in the GSI kinematic regime. To this end, we will apply the technique of threshold resummation, to which we now turn.

As we mentioned above,  $\tau$  is typically very large for the GSI kinematics,  $0.2 \lesssim \tau \lesssim 0.7$ . This is a region where higher-order corrections to the partonic cross sections are particularly important.

$\tau = 1$  sets a threshold for the reaction, and as  $\tau$  increases toward unity, very little phase space for real gluon radiation remains in the partonic process, since most of the initial partonic energy is used to produce the virtual photon. Virtual and real-emission diagrams then become strongly imbalanced, and the infrared cancellations leave behind large logarithmic higher order corrections to the partonic cross sections, the so-called threshold logarithms. At the  $k$ th order in perturbation theory, the leading logarithms are of the form  $\alpha_s^k \ln^{2k-1}(1-z)/(1-z)$ , where  $z = \tau/x_a x_b$  is the partonic analogue of  $\tau$ . For sufficiently large  $z$ , perturbative calculations to fixed order in  $\alpha_s$  become unreliable, since the double logarithms compensate the smallness of  $\alpha_s(M)$  even if  $M$  is of the order of a few GeV. The fact that the parton distributions are steeply falling functions of the momentum fractions  $x_{a,b}$  means that the threshold region is actually emphasized in the cross section, even if  $\tau$  itself is still rather far away from one. If  $\tau$  is close to unity, as is the case for much of the GSI kinematics, the region of large  $z \lesssim 1$  completely dominates, and it is crucial that the terms  $\alpha_s^k \ln^{2k-1}(1-z)/(1-z)$  be resummed to all orders in  $\alpha_s$ . Such a “threshold resummation” was originally developed for the Drell-Yan process [23, 24] and subsequently applied to a variety of more involved partonic processes in QCD<sup>†</sup>. It turns out that the soft-gluon effects exponentiate, not in  $z$ -space directly, but in Mellin- $N$  moment space, where  $N$  is the Mellin moment conjugate to  $z$ . The leading logarithms (LL) in the exponent are of the form  $\alpha_s^k \ln^{k+1}(N)$ , subleading logarithms (next-to-leading logarithms (NLL)) of  $\alpha_s^k \ln^k(N)$ . In this paper we will use NLL resummed perturbation theory to analyze the importance of higher-order corrections to the Drell-Yan cross section in the kinematic regime to be explored at the GSI.

There is a close relation between resummation and the nonperturbative power corrections. Taking into account nonleading logarithms and the running of the coupling, resummation always leads to a perturbative expression in which the scale of the coupling reflects the value of the transform variable. Because of the singularity of the perturbative effective coupling at  $\Lambda_{QCD}$ , the resulting expressions are ill-defined [28]. The analysis of these ambiguities for Drell-Yan cross sections [29, 30] suggests a series of nonperturbative corrections [29], generically suppressed by even powers of the pair mass  $M$ , but enhanced by the moment variable  $N$ ,  $N^2/M^2$ . As we shall see, for GSI fixed-target energies, the effective values of  $N$  are so large that the first few power corrections will not suffice for the Drell-Yan cross section. We will therefore rely on a somewhat different approach, to be presented in more detail elsewhere, and cut off unphysical dependence on low momentum scales. The result for a large portion of dilepton masses  $M$  will be a cross section with moderated, but still significant enhancements relative even to next-to-next-to-leading order calculations, which we take as a conservative prediction based on perturbation theory. Experimental results on these cross sections should shed light on the interrelations between fixed-order, all-order and nonperturbative corrections in hadronic scattering.

The remainder of this paper is organized as follows. Section 2 will present the basic framework for our calculations and will introduce the partonic threshold region. In sec. 3, we provide all ingredients for the NLL resummation of the threshold logarithms, and we also propose a new infrared-regulated expression for the form of nonperturbative corrections suggested by perturbative resummation. In sec. 4 we then present phenomenological results for the regions of interest in GSI measurements.

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<sup>†</sup>See, for example, the review in Ref. [25], and interesting recent work [26] that rederives some of these results in the context of the soft-collinear effective theory [27].

## 2 Perturbative cross section and the threshold region

The spin-dependent cross section for dilepton production by two transversely polarized hadrons is defined as

$$\frac{\tau d\delta\sigma}{d\tau d\phi} \equiv \frac{1}{2} \left( \frac{\tau d\sigma^{\uparrow\uparrow}}{d\tau d\phi} - \frac{\tau d\sigma^{\uparrow\downarrow}}{d\tau d\phi} \right), \quad (5)$$

where the superscript  $\uparrow\uparrow$  ( $\uparrow\downarrow$ ) denotes parallel (antiparallel) setting of the transverse spins of the incoming hadrons. We have used the customary Drell-Yan scaling variable  $\tau = M^2/S$  with  $M$  the invariant mass of the lepton pair and  $S$  the center-of-mass energy squared.  $\phi$  is the azimuthal angle of one of the leptons, counted relative to the axis defined by the transverse polarizations. For simplicity, we have integrated over all rapidities of the lepton pair. At high  $M$ , the cross section factorizes into convolutions of the transversity distributions  $\delta f$  with the corresponding transversely polarized partonic hard-scattering cross sections [7]:

$$\frac{\tau d\delta\sigma(\tau)}{d\tau d\phi} = \sum_{a,b} \sigma_{ab}^{(0)}(\phi) \int_{\tau}^1 \frac{dx_a}{x_a} \int_{\tau/x_a}^1 \frac{dx_b}{x_b} \delta f_a(x_a, \mu^2) \delta f_b(x_b, \mu^2) \delta\omega_{ab} \left( z \equiv \tau/x_a x_b, \frac{M^2}{\mu^2}, \alpha_s(\mu) \right), \quad (6)$$

where  $\mu$  collectively denotes the factorization and renormalization scales, and where we will specify the  $\sigma_{ab}^{(0)}(\phi)$  below. Due to the odd chirality of transversity, and since there is no transversity gluon density,  $q\bar{q}$  annihilation is the only partonic channel, up to next-to-leading order (NLO)<sup>‡</sup>. Its cross section is calculated in QCD perturbation theory as a series in  $\alpha_s$ :

$$\delta\omega_{ab}(z, r, \alpha_s) = \delta\omega_{ab}^{(0)}(z) + \frac{\alpha_s}{\pi} \delta\omega_{ab}^{(1)}(z, r) + \left( \frac{\alpha_s}{\pi} \right)^2 \delta\omega_{ab}^{(2)}(z, r) + \dots, \quad (7)$$

where  $r = M^2/\mu^2$ . Since we will not consider very high energies and dilepton masses, only photons contribute as intermediate particles. The lowest-order (LO,  $\mathcal{O}(\alpha_s^0)$ ) process thus is  $q^\uparrow \bar{q}^\uparrow \rightarrow \gamma^* \rightarrow l^+ l^-$ , for which

$$\delta\omega_{q\bar{q}}^{(0)}(z) = \delta(1-z), \quad \sigma_{q\bar{q}}^{(0)}(\phi) = \frac{\alpha^2 e_q^2}{9S} \cos(2\phi). \quad (8)$$

The first-order term  $\delta\omega_{q\bar{q}}^{(1)}$  is known and reads [31]

$$\begin{aligned} \delta\omega_{q\bar{q}}^{(1)}(z, r) = C_F \left[ 4z \left( \frac{\ln(1-z)}{1-z} \right)_+ - \frac{2z \ln z}{1-z} - \frac{3z \ln^2 z}{1-z} + 2(1-z) \right. \\ \left. + \left( \frac{\pi^2}{3} - 4 \right) \delta(1-z) + \left( \frac{2z}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right) \ln r \right], \end{aligned} \quad (9)$$

where  $C_F = 4/3$  and the “+”-distribution is defined as

$$\int_0^1 dz [g(z)]_+ f(z) = \int_0^1 dz g(z) (f(z) - f(1)). \quad (10)$$

Since resummation is performed in Mellin moment space, we take a Mellin transform of the hadronic cross section:

$$\frac{d\delta\sigma^N}{d\phi} \equiv \int_0^1 d\tau \tau^{N-1} \frac{\tau d\delta\sigma}{d\tau d\phi}. \quad (11)$$

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<sup>‡</sup>Starting from next-to-next-to-leading order (NNLO) there are contributions from  $qq$  scattering as well.

The cross section algebraically factorizes under moments,

$$\frac{d\delta\sigma^N}{d\phi} = \sigma_0 \sum_q \delta q^N(\mu^2) \delta \bar{q}^N(\mu^2) \left[ 1 + \frac{\alpha_s(\mu)}{\pi} \delta\omega_{q\bar{q}}^{(1),N}(r) + \left( \frac{\alpha_s(\mu)}{\pi} \right)^2 \delta\omega_{q\bar{q}}^{(2),N}(r) + \dots \right], \quad (12)$$

where the Mellin moments of the transversity distributions  $\delta q$  and the higher-order corrections  $\delta\omega_{q\bar{q}}^{(k)}$  are defined as usual,

$$\begin{aligned} \delta q^N(\mu^2) &= \int_0^1 dx x^{N-1} \delta f_q(x, \mu^2), \\ \delta\omega_{q\bar{q}}^{(k),N}(r) &= \int_0^1 dz z^{N-1} \delta\omega_{q\bar{q}}^{(k)}(z, r). \end{aligned} \quad (13)$$

The moments of the first-order correction  $\delta\omega_{q\bar{q}}^{(1)}$  in Eq. (9) read in the  $\overline{\text{MS}}$  scheme [31]:

$$\delta\omega_{q\bar{q}}^{(1),N}(r) = C_F \left[ \frac{2}{N(N+1)} + 2S_1^2(N) + 6(S_3(N) - \zeta(3)) - 4 + \frac{2}{3}\pi^2 + \left( \frac{3}{2} - 2S_1(N) \right) \ln r \right]. \quad (14)$$

The sums appearing here are defined by

$$S_k(N) \equiv \sum_{j=1}^N \frac{1}{j^k}. \quad (15)$$

Their analytic continuations to arbitrary Mellin- $N$  are

$$\begin{aligned} S_1(N) &= \psi(N+1) + \gamma_E, \\ S_3(N) &= \frac{1}{2}\psi''(N+1) + \zeta(3), \end{aligned} \quad (16)$$

where  $\psi(z)$  is the digamma function,  $\gamma_E = 0.5772\dots$  is the Euler constant, and  $\zeta(3) \approx 1.202057$ .

We mention that formulas analogous to the above hold for the unpolarized case. The main difference is that in the unpolarized case beyond LO there are contributions from initial-state gluons to the Drell-Yan cross section. In the kinematic region we are interested in here, these are rather unimportant. All details for the unpolarized case to NLO may for example be found in Ref. [9].

The threshold region corresponds to  $z \rightarrow 1$  or  $N \rightarrow \infty$ . At large  $N$ , the moments of the NLO correction become

$$\delta\omega_{q\bar{q}}^{(1),N}(r) = C_F \left[ 2\ln^2(\bar{N}) - 4 + \frac{2}{3}\pi^2 + \left( \frac{3}{2} - 2\ln(\bar{N}) \right) \ln r \right] + \mathcal{O}\left(\frac{1}{N}\right), \quad (17)$$

where

$$\bar{N} = N e^{\gamma_E}. \quad (18)$$

One can see the double-logarithmic corrections  $\propto \alpha_s \ln^2(\bar{N})$  near threshold, associated with the logarithmic term  $\sim \ln(1-z)/(1-z)$  in Eq. (9). At higher orders, there are corrections of the form  $\alpha_s^k \ln^l(\bar{N})$  with  $l \leq 2k$ . We emphasize that the behavior of the unpolarized partonic cross section near threshold is exactly the same as Eq. (17). This is related to the fact that the large logarithms

are due to the emission of soft gluons, which is spin-independent. Thanks to the simple structure of the LO Drell-Yan process, the constant ( $N$ -independent) pieces which are partly associated with virtual corrections are identical as well for the transversely polarized and unpolarized cases, provided both are treated in the same factorization scheme.

We now turn to the resummation of the leading and next-to-leading threshold logarithms to all orders in  $\alpha_s$ .

### 3 Resummed cross section

In Mellin-moment space, threshold resummation for the Drell-Yan process results in the exponentiation of the soft-gluon corrections. To NLL<sup>§</sup>, the resummed formula is given in the  $\overline{\text{MS}}$  scheme by

$$\delta\omega_{q\bar{q}}^{\text{res},N}(r, \alpha_s(\mu)) = \exp[C_q(r, \alpha_s(\mu))] \exp\left\{2 \int_0^1 dz \frac{z^{N-1} - 1}{1 - z} \int_{\mu^2}^{(1-z)^2 M^2} \frac{dk_T^2}{k_T^2} A_q(\alpha_s(k_T))\right\}, \quad (19)$$

where

$$A_q(\alpha_s) = \frac{\alpha_s}{\pi} A_q^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 A_q^{(2)} + \dots, \quad (20)$$

with [33]:

$$A_q^{(1)} = C_F, \quad A_q^{(2)} = \frac{1}{2} C_F \left[ C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} N_f \right], \quad (21)$$

where  $N_f$  is the number of flavors and  $C_A = 3$ . The coefficient  $C_q(r, \alpha_s(\mu))$  collects mostly hard virtual corrections. It is a perturbative series and reads

$$C_q(r, \alpha_s(\mu)) = \frac{\alpha_s}{\pi} C_F \left( -4 + \frac{2\pi^2}{3} + \frac{3}{2} \ln r \right) + \mathcal{O}(\alpha_s^2). \quad (22)$$

We note that it was shown in [34] that these corrections also exponentiate.

Eq. (19) as it stands is ill-defined because of the divergence in the perturbative running coupling  $\alpha_s(k_T)$  at  $k_T = \Lambda_{\text{QCD}}$ . The perturbative expansion of the expression shows factorial divergence, which in QCD corresponds to a power-like ambiguity of the series. It turns out, however, that the factorial divergence appears only at nonleading powers of momentum transfer. The large logarithms we are resumming arise in the region [24]  $z \leq 1 - 1/\bar{N}$  in the integrand in Eq. (19). One therefore finds that to NLL they are contained in the simpler expression

$$2 \int_{M^2/\bar{N}^2}^{M^2} \frac{dk_T^2}{k_T^2} A_q(\alpha_s(k_T)) \ln \frac{\bar{N} k_T}{M} + 2 \int_{M^2}^{\mu^2} \frac{dk_T^2}{k_T^2} A_q(\alpha_s(k_T)) \ln \bar{N} \quad (23)$$

for the second exponent in (19). This form, to which we will return below, is used for “minimal” expansions [35] of the resummed exponent.

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<sup>§</sup>We note that the threshold resummation for the Drell-Yan process has been worked out even to next-to-next-to-leading logarithmic accuracy, see [32].

### 3.1 Exponents at NLL

In the exponents, the large logarithms in  $N$  now occur only as single logarithms, of the form  $\alpha_s^k \ln^{k+1}(N)$  for the leading terms. Subleading terms are down by one or more powers of  $\ln(N)$ . Knowledge of the coefficients  $A_q^{(1,2)}$  in Eq. (19) is enough to resum the full towers of LL terms  $\alpha_s^k \ln^{k+1}(N)$ , and NLL ones  $\alpha_s^k \ln^k(N)$  in the exponent. With the coefficient  $C_q$  one then gains control of three towers of logarithms in the cross section,  $\alpha_s^k \ln^{2k}(N)$ ,  $\alpha_s^k \ln^{2k-1}(N)$ ,  $\alpha_s^k \ln^{2k-2}(N)$ .

We now give the explicit formula for the expansion of the resummed exponent to NLL accuracy. From Eqs. (19),(23) one finds [35, 36]

$$\ln \delta\omega_{q\bar{q}}^{\text{res},N}(r, \alpha_s(\mu)) = C_q(r, \alpha_s(\mu)) + 2 \ln \bar{N} h^{(1)}(\lambda) + 2h^{(2)}(\lambda, r), \quad (24)$$

where

$$\lambda = b_0 \alpha_s(\mu) \ln \bar{N}. \quad (25)$$

The functions  $h^{(1,2)}$  are given by

$$h^{(1)}(\lambda) = \frac{A_q^{(1)}}{2\pi b_0 \lambda} [2\lambda + (1 - 2\lambda) \ln(1 - 2\lambda)], \quad (26)$$

$$\begin{aligned} h^{(2)}(\lambda, r) = & -\frac{A_q^{(2)}}{2\pi^2 b_0^2} [2\lambda + \ln(1 - 2\lambda)] + \frac{A_q^{(1)} b_1}{2\pi b_0^3} \left[ 2\lambda + \ln(1 - 2\lambda) + \frac{1}{2} \ln^2(1 - 2\lambda) \right] \\ & + \frac{A_q^{(1)}}{2\pi b_0} [2\lambda + \ln(1 - 2\lambda)] \ln(r) - \frac{A_q^{(1)} \alpha_s(\mu)}{\pi} \ln \bar{N} \ln(r), \end{aligned} \quad (27)$$

where

$$\begin{aligned} b_0 &= \frac{1}{12\pi} (11C_A - 2N_f), \\ b_1 &= \frac{1}{24\pi^2} (17C_A^2 - 5C_A N_f - 3C_F N_f). \end{aligned} \quad (28)$$

The function  $h^{(1)}$  contains all LL terms in the perturbative series, while  $h^{(2)}$  is of NLL only. We note that the resummed exponent depends on the factorization scales in such a way that it will compensate the scale dependence (evolution) of the parton distributions. This feature is represented by the last term in (27). One therefore expects a decrease in scale dependence of the cross section from resummation. The remaining  $\mu$ -dependence in the second to last term in (27) results from writing the strong coupling constant as

$$\alpha_s(k_T) = \frac{\alpha_s(\mu)}{1 + b_0 \alpha_s(\mu) \ln(k_T^2/\mu^2)} + \mathcal{O}(\alpha_s(\mu)^2 (\alpha_s(\mu) \ln(k_T^2/\mu^2))^n) \quad (29)$$

when doing the NLL expansion of the exponent. For this term,  $\mu$  represents the renormalization scale.

As was shown in Refs. [30, 37], it is possible to improve the above formula slightly and to also correctly take into account certain subleading terms in the resummation. To this end, we rewrite



Eqs. (24)-(27) as

$$\begin{aligned}
\ln \delta\omega_{q\bar{q}}^{\text{res},N}(r, \alpha_s(\mu)) &= \frac{1}{\pi b_0} [2\lambda + \ln(1-2\lambda)] \left( \frac{A_q^{(1)}}{b_0 \alpha_s(\mu)} - \frac{A_q^{(2)}}{\pi b_0} + \frac{A_q^{(1)} b_1}{b_0^2} + A_q^{(1)} \ln r \right) \\
&+ \frac{\alpha_s}{\pi} C_F \left( -4 + \frac{2\pi^2}{3} \right) + \frac{A_q^{(1)} b_1}{2\pi b_0^3} \ln^2(1-2\lambda) + B_q^{(1)} \frac{\ln(1-2\lambda)}{\pi b_0} \\
&+ [-2A_q^{(1)} \ln \bar{N} - B_q^{(1)}] \left( \frac{\alpha_s(\mu)}{\pi} \ln r + \frac{\ln(1-2\lambda)}{\pi b_0} \right), \tag{30}
\end{aligned}$$

where  $B_q^{(1)} = -3C_F/2$ . The last term in Eq. (30) is the LL expansion of the term

$$\int_{\mu^2}^{M^2/\bar{N}^2} \frac{dk_T^2}{k_T^2} \frac{\alpha_s(k_T)}{\pi} [-2A_q^{(1)} \ln \bar{N} - B_q^{(1)}]. \tag{31}$$

Since the factor  $[-2A_q^{(1)} \ln \bar{N} - B_q^{(1)}]$  is the large- $N$  limit of the moments of the LO DGLAP splitting function for transversity,  $\delta P^N$ , the term in Eq. (31) may be viewed as an evolution of the parton distributions between scales  $\mu$  and  $M/\bar{N}$ . This suggests to modify the resummation by replacing [30, 38]

$$[-2A_q^{(1)} \ln \bar{N} - B_q^{(1)}] \longrightarrow \delta P^N \equiv C_F \left[ \frac{3}{2} - 2S_1(N) \right] \tag{32}$$

in Eq. (30). To see the improvement resulting from this, we expand the resummed formula in Eq. (30), after the replacement (32), to first order in  $\alpha_s(\mu)$  and find:

$$\delta\omega_{q\bar{q}}^{\text{res},N}(r, \alpha_s) = 1 + \frac{\alpha_s}{\pi} C_F \left[ 4 \ln(\bar{N}) S_1(N) - 2 \ln^2(\bar{N}) - 4 + \frac{2}{3} \pi^2 + \left( \frac{3}{2} - 2S_1(N) \right) \ln r \right] + \mathcal{O}(\alpha_s^2). \tag{33}$$

This term correctly gives the large- $N$  pieces of the NLO cross section in Eq. (17), but it goes beyond that by also reproducing all contributions  $\sim \ln(\bar{N})/N$  in Eq. (14), the latter arising from the expansion  $S_1(N) = \ln(\bar{N}) + 1/(2N) + \mathcal{O}(1/N^2)$ .

The unpolarized resummed partonic cross section in Mellin-moment space is practically identical to the transversely polarized one in Eq. (19), since the coefficients  $A(\alpha_s)$  and  $C_q(r, \alpha_s)$  are spin-independent and thus the same for the unpolarized and transversely polarized cases. A very small difference arises in the replacement in Eq. (32), in which for the unpolarized case one of course has to use the unpolarized LO splitting function. In addition, one should also take into account the singlet mixing in the evolution [30] which, however, is very small in the kinematic region we are interested in.

### 3.2 Far-infrared resummed cross section

We will see in the phenomenology section below that perturbative resummation as formulated so far in Eqs. (19) or (23) predicts very large enhancements of the lowest order cross section, sometimes by orders of magnitude. There is good reason to believe that this enhancement is

only partly physical. The large corrections arise from a region where the integral in the exponent becomes sensitive to the behavior of the integrand at small values of  $k_T$ . As long as  $\Lambda_{QCD} \ll M/\bar{N} \ll M$ , the use of NLL perturbation theory may be justified, but when  $|N|$  becomes so large that  $k_T$  goes down to nonperturbative scales, we may well question the self-consistency of perturbation theory. We now turn to the question of how this “far-infrared” limit should be treated in resummed perturbation theory. Our discussion here will be brief, and we will only present one model that addresses the far-infrared limit. A more detailed study of this very interesting regime will be presented in a future publication.

We seek a modification of the perturbative expression Eq. (19) that excludes the region in which the absolute value of  $k_T$  is less than some scale  $\mu_0 > m_\pi$ . We think of  $\mu_0$  as the scale beyond which the true mass spectrum of QCD replaces perturbation theory, regulating all soft and collinear singularities, so that  $m_\pi$  should be thought of only as a lower limit for  $\mu_0$ . To implement this idea, we will adopt a modified resummed hard scattering, which reproduces NLL logarithmic behavior in the moment variable  $N$  so long as  $M/\bar{N} > \mu_0$ , but “freezes” once  $M/\bar{N} < \mu_0$ . If nothing else, this will test the importance of the region  $k_T \leq \Lambda_{QCD}$  for the resummed cross section. If  $N$  were real and positive, we could simply replace the first exponent in (23) by

$$4 \int_{\rho(M/\bar{N}, \mu_0)}^M \frac{dk_T}{k_T} A_q(\alpha_s(k_T)) \ln \frac{\bar{N}k_T}{M}, \quad (34)$$

where

$$\rho(a, b) = \max(a, b), \quad (35)$$

and where  $\mu_0$  then serves to cut off the lower logarithmic behavior. To provide an expression that can be continued to complex  $N$ , we choose

$$\rho(a, b) = (a^p + b^p)^{1/p}, \quad (36)$$

with integer  $p$ . This simple form is consistent with the minimal expansion given above, and it also allows for a straightforward analysis of the ensuing branch cuts in the complex- $N$  plane. Vanishing  $\mu_0$  corresponds to the standard minimal form (23). For definiteness, and for simplicity, we choose  $p = 2$  in this paper. We will continue to use the expansions in Eq. (24), but redefining  $\lambda$  in Eq. (25) by

$$\lambda = b_0 \alpha_s(\mu) \ln \bar{N} - \frac{1}{2} b_0 \alpha_s(\mu) \ln \left( 1 + \frac{\bar{N}^2 \mu_0^2}{M^2} \right). \quad (37)$$

This form has the advantage that it generates only even power corrections in the ratio  $N^2/M^2$ , when this quantity is not too large [30]. We will investigate below the moderation of the perturbative increase provided by the cut-off  $\mu_0$ .

### 3.3 Matching to the NLO cross section, and inverse Mellin transform

As we have discussed above, the resummation is achieved in Mellin moment space. In order to obtain a resummed cross section in  $z$  space, one needs an inverse Mellin transform. This requires a prescription for dealing with the singularity in the perturbative strong coupling constant in Eqs. (19),(23) or in the NLL expansion, Eqs. (26),(27). We will use the Minimal Prescription developed in Ref. [35], which relies on use of the NLL expanded form Eqs. (26),(27), and on

choosing an appropriate contour in the complex- $N$  plane. In the standard minimal prescription, based on Eqs. (23) and (25), this contour is chosen to lie to the *left* of the “Landau” singularities at  $\lambda = 1/2$  in the Mellin integrand, which are far to the right in the  $N$  plane. With the modified variable  $\lambda$  of Eq. (37), however, branch cuts from  $\lambda = 1/2$  reside on the imaginary axis in the  $N$ -plane (and so do branch cuts that arise when the argument of the logarithm in Eq. (37) vanishes). We again choose our inverse contour as

$$\frac{\tau d\delta\sigma^{(\text{res})}}{d\tau d\phi} = \int_{C_{MP}-i\infty}^{C_{MP}+i\infty} \frac{dN}{2\pi i} \tau^{-N} \frac{d\delta\sigma^{(\text{res}),N}}{d\phi}, \quad (38)$$

where  $C_{MP}$  is any positive number. All singularities are then to the left of the contour. We keep the Mellin contour parallel to the imaginary- $N$  axis. The result defined by the minimal prescription has the property that its perturbative expansion is an asymptotic series that has no factorial divergence and therefore no “built-in” power-like ambiguities. Power corrections may then be added, as phenomenologically required, and in a sense our cut-off prescription does exactly this.

When performing the resummation, one of course wants to make full use of the available fixed-order cross section, which in our case is NLO ( $\mathcal{O}(\alpha_s)$ ). Therefore, a matching to this cross section is appropriate, which may be achieved by expanding the resummed cross section to  $\mathcal{O}(\alpha_s)$ , subtracting the expanded result from the resummed one, and adding the full NLO cross section:

$$\begin{aligned} \frac{\tau d\delta\sigma^{(\text{match})}}{d\tau d\phi} &= \sum_{q,\bar{q}} \sigma_{q\bar{q}}^{(0)}(\phi) \int_{C_{MP}-i\infty}^{C_{MP}+i\infty} \frac{dN}{2\pi i} \tau^{-N} \delta q^N(\mu^2) \delta \bar{q}^N(\mu^2) \\ &\times \left[ \delta\omega_{q\bar{q}}^{\text{res},N}(r, \alpha_s(\mu)) - \delta\omega_{q\bar{q}}^{\text{res},N}(r, \alpha_s(\mu)) \Big|_{\mathcal{O}(\alpha_s)} \right] + \frac{\tau d\delta\sigma^{(\text{NLO})}}{d\tau d\phi}. \end{aligned} \quad (39)$$

In this way, NLO is taken into account in full, and the soft-gluon contributions beyond NLO are resummed to NLL. Any double-counting of perturbative orders is avoided. As we will see below, however, matching is almost academic in the present calculation since the NLO-expansion of the resummed cross section agrees to within 0.1% with the full NLO one for the kinematics relevant here. We also note that whenever we will use the form (37) with a non-vanishing cut-off  $\mu_0$ , we will perform the matching using the expansion  $\delta\omega_{q\bar{q}}^{\text{res},N}(r, \alpha_s(\mu)) \Big|_{\mathcal{O}(\alpha_s)}$  in (39) evaluated at  $\mu_0 = 0$ .

## 4 Phenomenological Results

Starting from Eq. (39), we are now ready to present some first resummed results at the hadronic level. This is not meant to be an exhaustive study; rather we should like to investigate the overall size and relevance of the resummation effects. For this reason, we only consider the cross section  $d^2\sigma/dM d\phi$ , integrated over all rapidities. This should be sufficient to study the main effects. In experiment one will eventually study rapidity distributions in order to better pin down the  $x$ -dependences of the transversity densities. Our resummation could be extended to this case using techniques developed in [39]. We also note that the charmonium resonances will dominate over a part of the spectrum that we will consider. For our case study we will just ignore this, but remind the reader that a complete treatment of the dilepton spectrum will eventually also require the incorporation of charmonium production and its resummation effects.

We will investigate  $\bar{p}p$  collisions at four different energies. Each of these may eventually be realized at the GSI. The first two are in the fixed-target mode, relevant for the initial stage of  $\bar{p}p$  physics at the GSI, when antiprotons will be scattered off proton targets at energies in the range  $15 \text{ GeV} \lesssim E_{\bar{p}} \lesssim 25 \text{ GeV}$ . For definiteness, we will consider  $S = 30 \text{ GeV}^2$  and  $S = 45 \text{ GeV}^2$ . This is also the regime in which the theoretical description is most challenging. The second regime is for an asymmetric collider, with  $E_p = 3.5 \text{ GeV}$  and  $E_{\bar{p}} = 15 \text{ GeV}$ , corresponding to  $\sqrt{S} = 14.5 \text{ GeV}$ . Finally, we will consider a symmetric collider with  $E_p = E_{\bar{p}} = 15 \text{ GeV}$  ( $\sqrt{S} = 30 \text{ GeV}$ ). In all calculations below, we choose the renormalization and factorization scales as  $\mu = M$ .

## 4.1 Unpolarized cross section near partonic threshold

We start by considering the unpolarized cross section. For some of the kinematic regions described above, Drell-Yan experiments at the GSI would enter uncharted territory, and it will be crucial to develop confidence that the theoretical framework is understood. An important test would be a comparison to precise measurements of the unpolarized cross section.

For our unpolarized calculations we will use the NLO ( $\overline{\text{MS}}$  scheme) GRV parton distributions throughout [40]. In the unpolarized case, we are in the fortunate situation that even the full NNLO corrections to the partonic Drell-Yan cross section are available [41], which we will incorporate in our studies. These should in principle be used in conjunction with a set of NNLO parton distributions, which became available recently [42] after the computation of the three-loop evolution kernels [43]. The main purpose of our present studies, however, is to see how well the soft-gluon terms we are resumming reproduce also the NNLO corrections to the cross section. For this it is sufficient to stick to our use of NLO parton densities.

Figure 1 shows the effects of the higher orders generated by resummation, for the fixed-target cases with  $S = 30 \text{ GeV}^2$  and  $S = 45 \text{ GeV}^2$ . We define a resummed “ $K$ -factor” as the ratio of the resummed (matched) cross section to the LO cross section,

$$K^{(\text{res})} = \frac{d\sigma^{(\text{match})}/dM d\phi}{d\sigma^{(\text{LO})}/dM d\phi}, \quad (40)$$

which is shown by the solid line in Fig. 1. As can be seen,  $K^{(\text{res})}$  is very large, meaning that resummation results in a dramatic enhancement over LO, sometimes by over two orders of magnitude. It is then interesting to see how this enhancement builds up order by order in the resummed cross section. We expand the matched resummed formula to NLO and beyond and define the (here, not matched) “soft-gluon  $K$ -factors”

$$K^n \equiv \frac{d\sigma^{(\text{res})}/dM d\phi|_{\mathcal{O}(\alpha_s^n)}}{d\sigma^{(\text{LO})}/dM d\phi}, \quad (41)$$

which for  $n = 1, 2, \dots$  give the additional enhancement due to the  $\mathcal{O}(\alpha_s^n)$  terms in the resummed formula. Formally,  $K^0 = 1$ , while  $K^\infty = K^{(\text{res})}$  up to the effects of matching at NLO. The results for  $K^{1,2,3,4,6,8}$  are also shown in Fig. 1. One can see that there are very large contributions even beyond NNLO, in particular at the higher  $M$ . Clearly, the full resummation given by the solid line receives contributions from high orders. The symbols in Fig. 1 show the associated  $K$  factors for the exact NLO and NNLO calculations, respectively. One can see that these agree extremely

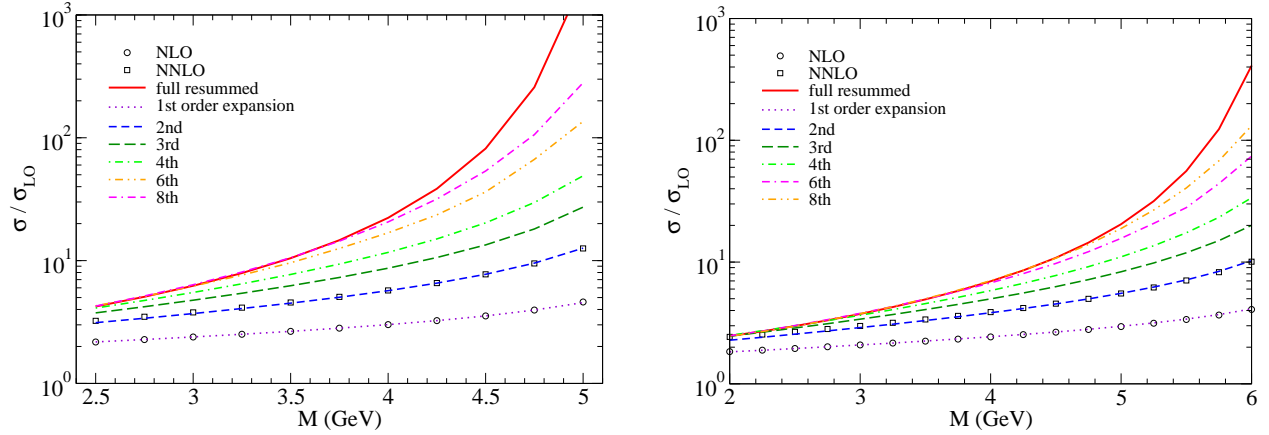


Figure 1: “ $K$ -factors” relative to LO as defined in Eqs. (40) and (41) for the Drell-Yan cross section in fixed-target  $\bar{p}p$  collisions at  $S = 30 \text{ GeV}^2$  (left) and  $S = 45 \text{ GeV}^2$  (right), as functions of lepton pair invariant mass  $M$ . The symbols denote the results for the exact NLO and NNLO calculations.

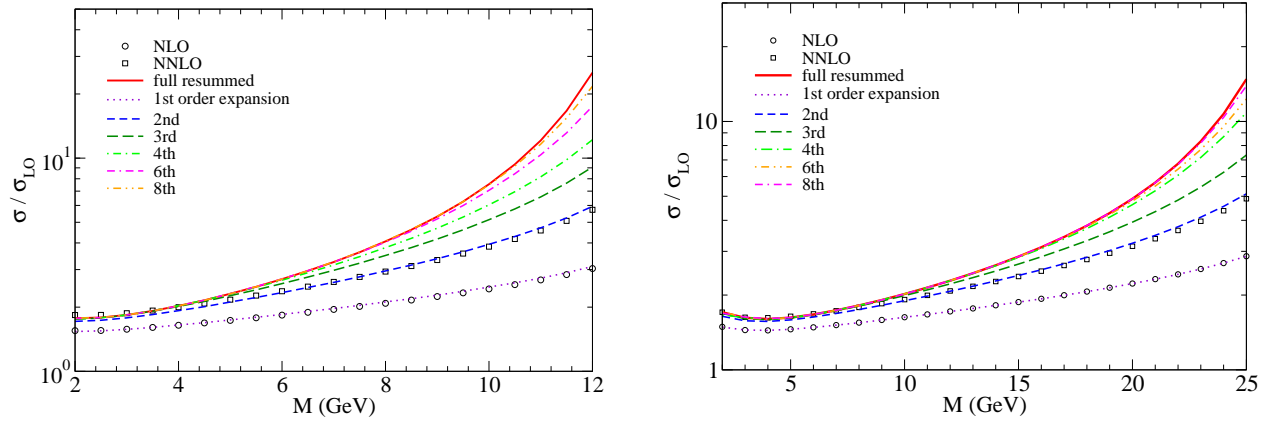


Figure 2: Same as Fig. 1, but for  $\bar{p}p$  collider options with  $\sqrt{S} = 14.5 \text{ GeV}$  (left) and  $\sqrt{S} = 30 \text{ GeV}$  (right).

well with the  $\mathcal{O}(\alpha_s)$  and  $\mathcal{O}(\alpha_s^2)$  expansions of the resummed cross section. In fact, the agreement between the full NLO result and the  $\mathcal{O}(\alpha_s)$  expansion of the resummed cross section is better than 0.1% over the whole range in  $M$  shown. Thus the matched resummed  $K$ -factor  $K^{(\text{res})}$  of Eq. (40) is also numerically very close to  $K^\infty$  as defined in (41). We note that the replacement in Eq. (32) helps somewhat in this comparison, in particular at NNLO it leads to a relative improvement of a few per cent. From the comparison we may conclude that the terms that we resum to all orders strongly dominate the cross section.

In Fig. 2, we show similar results for the two collider modes at  $\sqrt{S} = 14.5$  and  $30 \text{ GeV}$ . One can see that for a fixed Drell-Yan mass  $M$  the corrections become much smaller, since one is much further away from partonic threshold than for the fixed-target cases. Also, the convergence of the perturbative series occurs somewhat more rapidly. As before, the agreement between the  $\mathcal{O}(\alpha_s)$

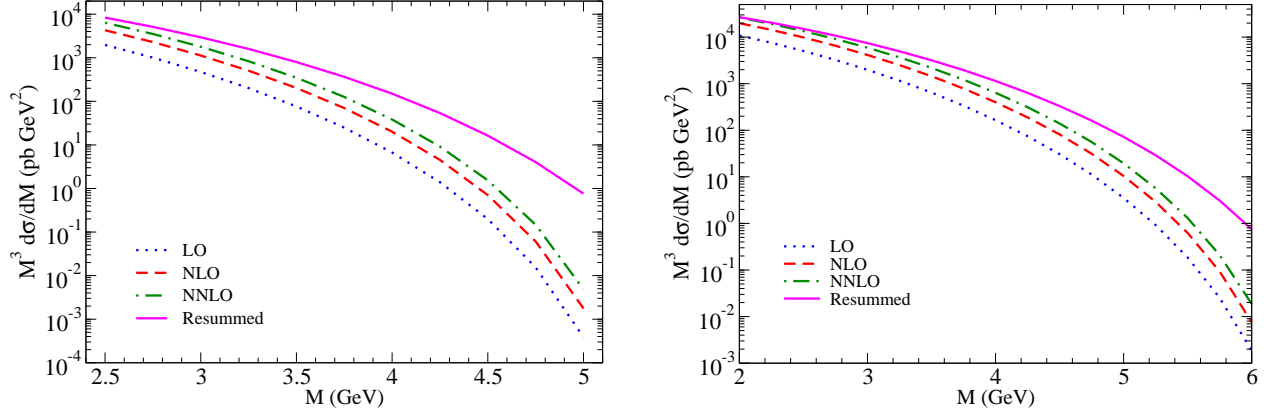


Figure 3: Unpolarized cross sections  $d^2\sigma/dM$  at  $S = 30 \text{ GeV}^2$  (left) and  $S = 45 \text{ GeV}^2$  (right) at LO, NLO, NNLO, and NLL resummed, as functions of lepton pair invariant mass  $M$ .

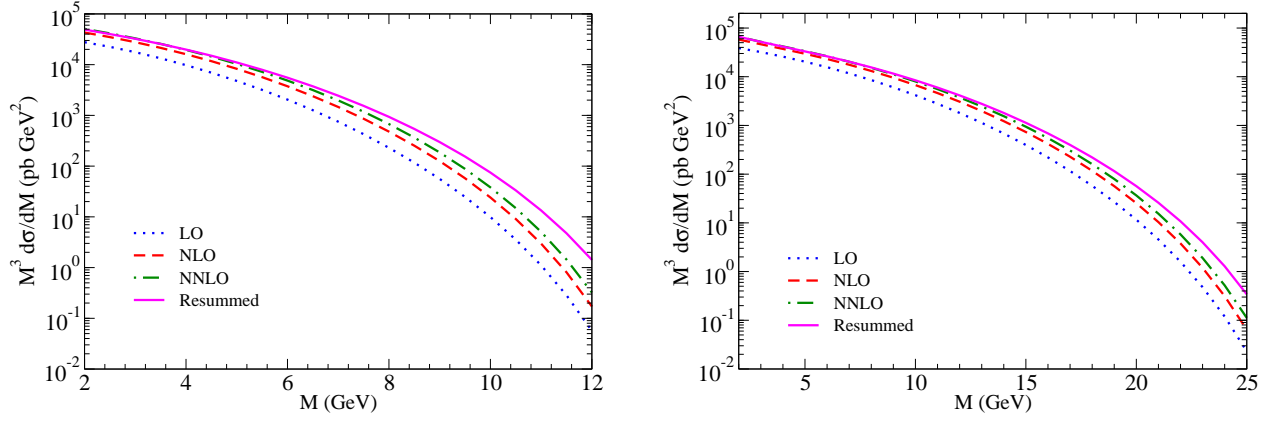


Figure 4: Same as Fig. 3, but for  $\bar{p}p$  collider options with  $\sqrt{S} = 14.5 \text{ GeV}$  (left) and  $\sqrt{S} = 30 \text{ GeV}$  (right).

and  $\mathcal{O}(\alpha_s^2)$  expansions of the resummed cross section and the exact NLO and NNLO calculations is very good, demonstrating the relevance of the resummed result.

In Figs. 3 and 4 we show the actual unpolarized cross sections  $M^3 d\sigma/dM$  corresponding to the results shown in Figs. 1 and 2, at (exact) fixed orders (LO, NLO, NNLO) in perturbation theory, and for the NLL resummed case. Here we have integrated over all azimuthal angles  $\phi$ <sup>¶</sup>.

One may wonder whether any sign of the need for large corrections beyond NLO can be found in previous Drell-Yan data [44]. The measurements at the lowest  $\sqrt{S}$  we are aware of are from the CERN WA39 [45] experiment and were made in  $\pi^\pm$ -Tungsten scattering. The pion energy was  $E_\pi = 39.5 \text{ GeV}$ , higher than what is considered for the fixed-target mode at the GSI, but still quite far below the collider energies. Fig. 5 shows the data, along with our results at LO, NLO, NNLO and NLL-resummed. We are using the parton distributions for the pion of [46]. It is

<sup>¶</sup>Note that the normalization factor  $\sigma_{q\bar{q}}^{(0)}$  in Eq. (8) becomes  $2\alpha^2 e_q^2/9S$  in the unpolarized case.

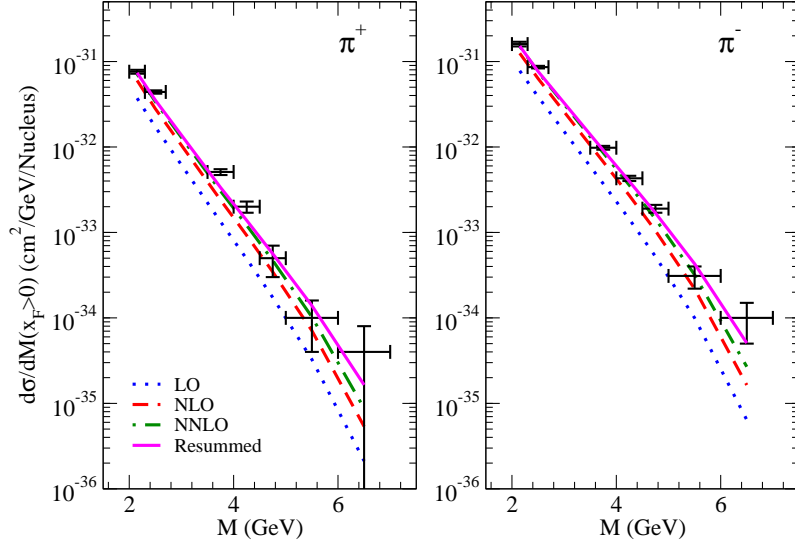


Figure 5: Comparison of LO, NLO, NNLO and resummed results to the WA39 data [45] for Drell-Yan dimuon production in  $\pi^\pm W$  scattering.

hard to draw definite conclusions from the comparison in Fig. 5, partly because the experimental uncertainties are rather large, and also because the pion parton densities are not known accurately. Nevertheless, it is interesting to observe that the data points at the highest dimuon mass  $M$  are quite consistent with large perturbative resummation effects.

Given the large, not to say huge, size of some of the enhancements, we now turn to the same ratios computed using the exponent (34) with the modified lower limit on the  $k_T$  integral, which regulates its infrared behavior. For simplicity, we will only show results for relatively small values of the cut-off scale  $\mu_0$  in (37). Of course, different choices give different results, but we should think of  $\mu_0$  as a kind of factorization scale, separating perturbative contributions from nonperturbative. Thus changes in  $\mu_0$  would be compensated at least in part by changes in a nonperturbative function. On the other hand, a very strong sensitivity to  $\mu_0$  can reasonably be interpreted as indicating that perturbative resummation alone cannot give a reliable estimate for the cross section. Our interest here, therefore, is primarily to illustrate the modification of the perturbative sector, which we do by choosing  $\mu_0 = 0.3$  GeV and 0.4 GeV. Results for the “ $K$ -factor” with these values of  $\mu_0$  are shown in Figs. 6 and 7, compared to the same NLO, NNLO and resummed factors shown above at the relevant energies and pair masses  $M$ . We hope to study the  $\mu_0$  dependence more extensively elsewhere, in connection with possible nonperturbative corrections.

The ratios of the new resummed but infrared-regulated cross sections to the LO one show a smoother increase than the pure minimal resummed cross sections. This difference is particularly marked at the lower center-of-mass energies in Fig. 6, with only a modest enhancement over NNLO remaining at  $\mu_0 = 0.3$  GeV, and even lower at 0.4 GeV. We note that the NLO and NNLO expansions of the resummed cross section turn out to be much less affected by the cut-off  $\mu_0$  than the full resummed cross section. At the higher energies of Fig. 7, the regulated resummed curves follow the unregulated curves far above NNLO, with much reduced sensitivity to  $\mu_0$ . We interpret

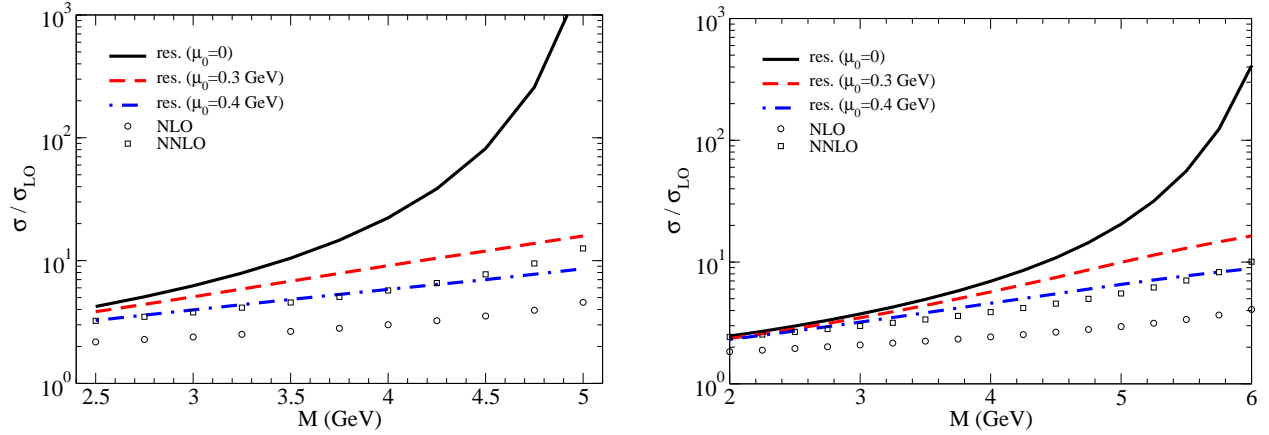


Figure 6: “ $K$ -factors” relative to LO as in Fig. 1, at  $S = 30 \text{ GeV}^2$  (left) and  $S = 45 \text{ GeV}^2$  (right). The dashed lines show the effect of a lower cut-off  $\mu_0 = 300 \text{ MeV}$  for the  $k_T$ -integral in the exponent, and the dot-dashed a cut-off of  $\mu_0 = 400 \text{ MeV}$  for comparison. The symbols denote the results for the exact NLO and NNLO calculations.

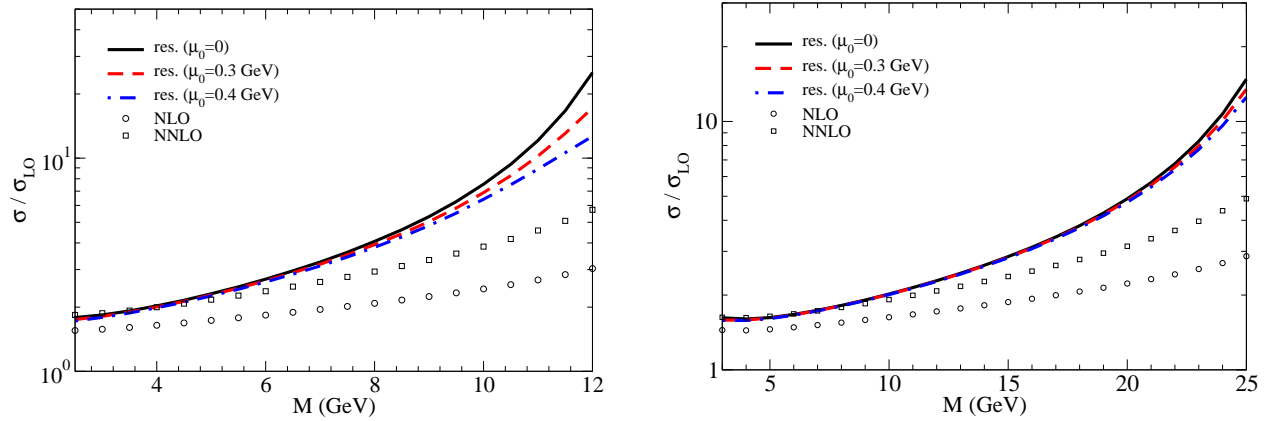


Figure 7: Same as Fig. 6, but for  $\bar{p}p$  collider options with  $\sqrt{S} = 14.5 \text{ GeV}$  (left) and  $\sqrt{S} = 30 \text{ GeV}$  (right).

these results to indicate a strong sensitivity to nonperturbative dynamics at the lower energies, and much less at the higher. We therefore take the predictions of large enhancements due to high order perturbation theory more seriously in the latter case, even while keeping in mind the WA39 measurements of Fig. 5 above, which suggest that large corrections, perturbative or not, should not be ruled out a priori. Data over the entire kinematic regime considered here would certainly shed a unique light on the transition between long- and short-distance effects in hadronic scattering.



## 4.2 Spin asymmetry $A_{TT}$

Before we can perform numerical studies of  $A_{TT}$  we need to make a model for the transversity densities in the valence region. Here, guidance is provided by the Soffer inequality [47]

$$2 |\delta q(x, Q^2)| \leq q(x, Q^2) + \Delta q(x, Q^2), \quad (42)$$

which gives an upper bound for each  $\delta q$ . Following [13, 14] we utilize this inequality by saturating the bound at some low input scale  $Q_0 \simeq 0.6 \text{ GeV}$  using the NLO GRV [40] and GRSV (“standard scenario”) [48] densities  $q(x, Q_0^2)$  and  $\Delta q(x, Q_0^2)$ , respectively. For  $Q > Q_0$  the transversity densities  $\delta q(x, Q^2)$  are then obtained by solving the evolution equations with the NLO [31, 49] evolution kernels. We refer the reader to [13, 14] for more details on our model distributions.

We will now investigate to what extent the large perturbative corrections we found for the unpolarized Drell-Yan cross section cancel in the spin asymmetry  $A_{TT}$ , Eq. (2). We have mentioned earlier that the resummation factors (19) for the  $q\bar{q}$  cross section are the same in the unpolarized and transversely polarized cases if both are treated in the same factorization scheme. Resummation effects would therefore cancel if the spin asymmetry were in Mellin-moment space. The convolution with the parton distributions and the inverse Mellin transform will affect the cancellation somewhat, but one still expects the spin asymmetry to be very robust. Indeed, the results for  $A_{TT}$  at LO, NLO, and resummed to NLL (with and without the cut-off  $\mu_0$ ), shown in Figs. 8 and 9 for the four energies that we consider, confirm this. We show

$$A_{TT} = \frac{d\delta\sigma/dM d\phi}{d\sigma/dM d\phi} \quad (43)$$

as a function of  $M$ . For simplicity we set  $\phi = 0$  here; extension to other  $\phi$  is straightforward by taking into account the  $\cos(2\phi)$ -dependence displayed in Eq. (8).

Aside from a slight deficit in the curves at lower  $M$  (and slight excess at higher  $M$ ) with unmodified minimal resummation at the lower  $S$ , all curves, including the LO, NLO and regulated resummed asymmetries all lie within a few percent of each other. We note that to NLO this robustness of  $A_{TT}$  was also found for  $p\bar{p}$  collisions at higher energies [50].

We can shed some light on why the asymmetry is modestly but significantly shifted for the unregulated perturbative resummed cross section with respect to the other cases shown. First, we observe that smaller momentum fractions  $x_{a,b}$  in the factorized cross section are associated with the cross section at smaller pair mass  $M$ , where the spin asymmetry is slightly smaller. Therefore, any effect that tends to drive momentum fractions particularly close to their minimum values, at  $z = 1$ , will tend to decrease the asymmetry. We note, however, that because the valence quark distributions are decreasing functions of the  $x$ ’s, and because they are the same for both hadrons, we might anticipate that the average values  $\langle x_a \rangle = \langle x_b \rangle$  are not far from their symmetric values,  $x_a = x_b = \sqrt{\tau}$  at partonic threshold. In fact, this turns out to be a surprisingly accurate estimate, even at lowest order, as can be readily verified from Fig. 10, where we plot  $\langle x \rangle$  for LO, NLO, and the unregulated ( $\mu_0 = 0$ ) and regulated ( $\mu_0 = 300 \text{ MeV}$ ) resummed cross sections. In this figure  $\langle x \rangle$  is found by performing the integral for the unpolarized cross section with an extra factor  $x_a$  in the integrand, and by dividing by the cross section itself.

Beyond lowest order, the hard-scattering cross section is further enhanced at partonic threshold ( $z = 1$ ), and we would expect that large perturbative enhancements, such as those associated

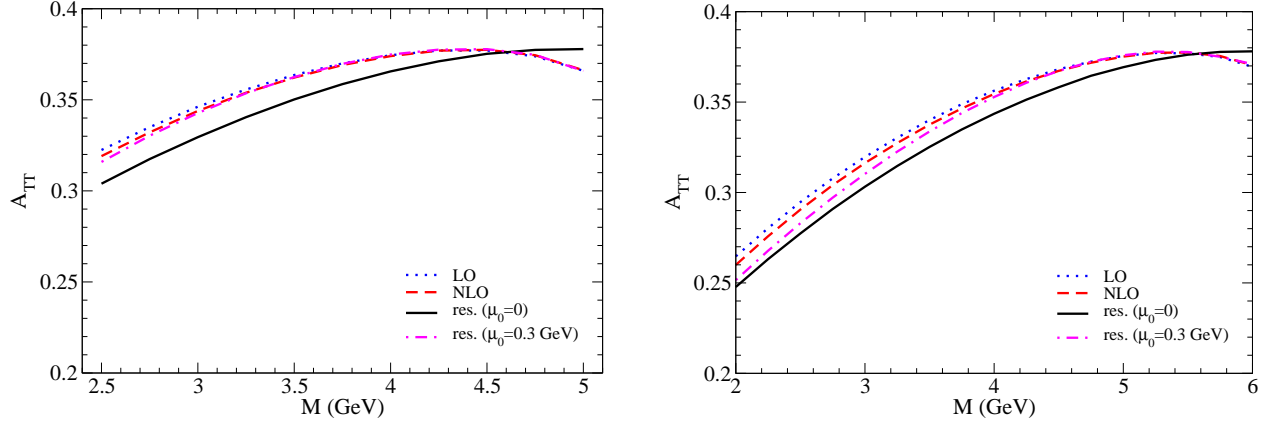


Figure 8: Spin asymmetry  $A_{TT}(\phi = 0)$  at LO, NLO and for the NLL resummed case, at  $S = 30 \text{ GeV}^2$  (left) and  $S = 45 \text{ GeV}^2$  (right). The dash-dotted lines show the effect of a lower cut-off  $\mu_0 = 300 \text{ MeV}$  for the  $k_T$ -integral in the exponent.

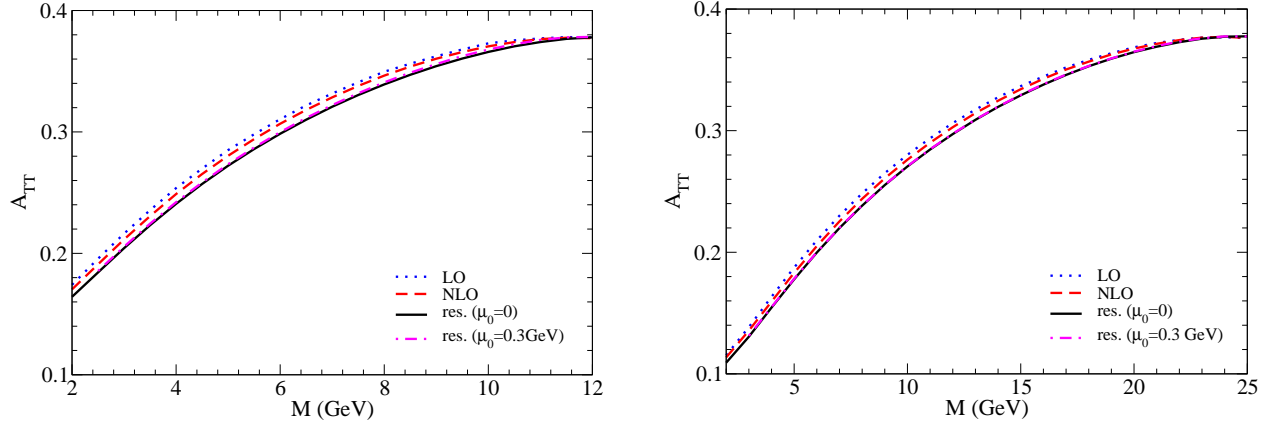


Figure 9: Same as Fig. 8, but for  $p\bar{p}$  collider options with  $\sqrt{S} = 14.5 \text{ GeV}$  (left) and  $\sqrt{S} = 30 \text{ GeV}$  (right).

with the plus distribution of Eq. (9), would force the average partonic center-of-mass energy still closer to threshold, and hence reduce the asymmetry even further. We should observe, however, that the hard scattering function is not a positive-definite cross section, but rather a sum of plus distributions, given at first nontrivial order in Eq. (9). At NLO, for example, the positive contribution at  $z = 1$  is from a delta function associated with virtual corrections, while the real-gluon contribution,  $\ln(1 - z)/(1 - z)$ , is actually negative, due to the subtraction of collinear divergences in the calculation of the hard scattering [23, 24]. We therefore cannot interpret  $\langle x_{a,b} \rangle$  as averages in the usual sense. In any case, we do see a more significant decrease in  $\langle x \rangle$ , computed above, for the unregulated resummed cross section than for fixed order. In fact, the values derived in this manner are below  $\sqrt{\tau}$ , which would be the lower limit for a positive-definite hard scattering function, and at the highest  $M$ , even below  $\tau$ , which is the lower limit of the integration range for the  $x$ 's. The caveat against a literal interpretation of  $\langle x \rangle$  notwithstanding, it is reasonable to interpret the modest decrease in the asymmetry for the unregulated resummed cross section as

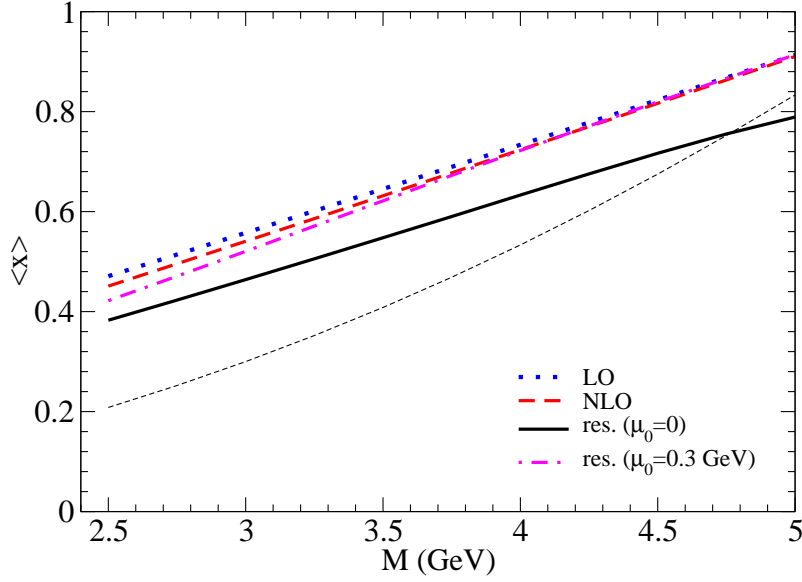


Figure 10: Average values  $\langle x_a \rangle = \langle x_b \rangle \equiv \langle x \rangle$  for the parton momentum fractions at  $S = 30 \text{ GeV}^2$  as functions of  $M$ , at LO, NLO, for purely perturbative resummed, and for regulated resummed with  $\mu_0 = 300 \text{ MeV}$ . For comparison, the lowest curve shows the corresponding values of  $\tau = M^2/S$ . A curve for  $\sqrt{\tau}$  – a straight line in this plot – would be practically indistinguishable from the LO one, although remaining slightly below it.

resulting from a hard-scattering function that is exceptionally peaked near  $z = 1$  in this case.

### 4.3 Use of DIS parton distribution functions

We recall that throughout this study, we have used the NLO parton distribution functions of Ref. [40]. Especially given the large effects we have found with resummation, we should revisit the use of “un-resummed” parton distributions in our study. The momentum fractions probed in the parton distributions become very large in the fixed-target regime, as shown by Fig. 10. One may wonder here to what extent the parton distributions themselves should include resummation effects. The densities we use have been determined mostly from an analysis of data from deeply-inelastic scattering (DIS), in which however no resummation of large- $N$  logarithms was included. It is known that soft-gluon resummation effects in DIS are rather unimportant, except at moderate photon virtuality  $Q^2$  and very large  $x$  [51, 52]. Nonetheless, we will give a rough estimate of the quantitative effect on the Drell-Yan cross section that might occur *if* one used parton densities determined from an analysis of the DIS data including resummation. We follow [51] to determine a model set of “ $\overline{\text{MS}}$ -resummed” valence distributions  $q^{N,\text{res}}$  (in Mellin-moment space) by demanding that their contributions to the DIS structure function  $F_2$  match those of the corresponding NLO densities at a fixed scale  $Q$ . This is ensured by “rescaling” the parton densities:

$$q^{N,\text{res}}(Q^2) = q^{N,\text{NLO}}(Q^2) \frac{C_2^{\text{NLO}}(N, Q^2)}{C_2^{\text{res}}(N, Q^2)}, \quad (44)$$

where  $C_2^{\text{NLO}}$  and  $C_2^{\text{res}}$  are the perturbative NLO and NLL resummed quark coefficient functions for  $F_2$ , respectively, which may be found in [51] for example. We choose  $Q$  in Eq. (44) fairly large, so that it is in a region where resummation effects are expected to be small and NLO to yield a good description of  $F_2$ . For illustration, we use two different values,  $Q^2 = 25 \text{ GeV}^2$  and  $Q^2 = 100 \text{ GeV}^2$ . The ansatz (44) then represents an estimate of the likely change in the parton distributions from resummation in DIS, and the parton densities  $q^{N,\text{res}}$  may be used for calculating the Drell-Yan cross section. The result is shown in Fig. 11, where we repeat the resummed cross section from Fig. 3, and display the effects on the cross section for the two choices of  $Q$ . A moderate decrease of the resummed cross section is found. We note that this effect becomes smaller at the collider energies, for a given  $M$ .

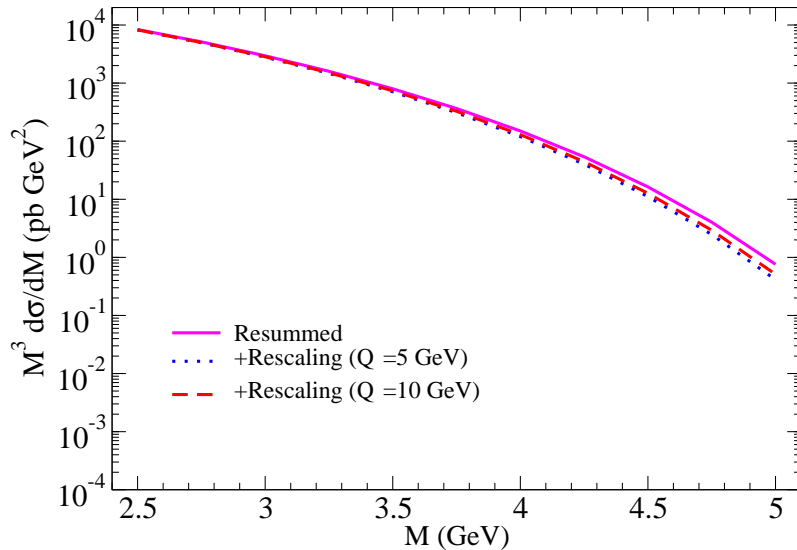


Figure 11: Effect on the unpolarized Drell-Yan cross section at  $S = 30 \text{ GeV}^2$  due to “rescaling” of the parton distributions as in Eq. (44). The solid line shows the result for the resummed cross section as in Fig. 3, the dotted and dashed lines are for  $Q = 5$  and  $10 \text{ GeV}$ , respectively, in Eq. (44).

## 5 Conclusions

We have verified that perturbative corrections associated with partonic threshold are large for dilepton production in the kinematic region being considered for proton-antiproton collisions at GSI. The close agreement between the fixed-order expansions of threshold-resummed cross sections and exact fixed order calculations suggests that the resummation is physically relevant, and significantly enhances the unpolarized Drell-Yan cross sections beyond fixed orders. At the same time, the transverse-spin asymmetries whose measurement is suggested for these experiments, are remarkably insensitive to shifts in the overall normalization. In summary, perturbative corrections appear to make the cross sections larger independently of spin. They would therefore make easier the study of spin asymmetries, and ultimately transversity distributions.

We have also shown that at the lower energies considered, the resummed cross section decreases markedly when an infrared cut-off is used to regulate contributions from soft gluon emission in the far infrared region. We have shown how to incorporate such a cut-off as a generalization of minimal resummation. At the lower energies, a strong sensitivity to the cut-off suggests that the large enhancements found with unregulated perturbative resummation arise from an unwarranted extension of perturbation theory into the soft region. At the same time, it is interesting to note that quite substantial  $K$ -factors survive almost unchanged by infrared regulation at the higher energies considered. This suggests that the measurement of the unpolarized dilepton cross section will shed light on the relationship between fixed orders, perturbative resummation and nonperturbative dynamics in hadronic scattering.

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